Idea 2: Newtonian Mechanics and Causality

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1 Preamble: Earthly Laws and Determinism

Our **first idea** told us that there was something at work in the Cosmos other than random chance or the capriciousness of the gods. (It did not tell us that there were not gods, but rather that their was not angry and randomly intervening gods.) In the first Idea, we discovered **laws**.

We learned that the heavens, the domain of divinity for most people throughout history, could be predicted. In our first idea, a specific analytic guiding principle that could be tested and used for completely reliable prediction was good enough to be called a "law". We are eliminating a lot of other very reliable predictive capabilities mankind had developed. Things like planting by the light of the moon and blood letting to remove bad humors were pretty good laws by the standards of 500 years ago.

Our second idea is going to up the ante on what constitutes a law. We are going to ask for things that are even more succinct and more simple and explain more. We raise our standards, but we will now call these "laws of nature". In a short time, we will decide that Kepler's Laws are not truly laws of nature, but convenient rules that are the consequence of Newton's laws.

2 Aristotle's Explanation of Mechanics

History records that Aristotle had theories on what seems like most of the physical world and well as politics and human behavior. In the arena of mechanics, we have already discussed the nature of Earthly motion and order in the context of the four basic elements, earth air, fire and water. Actually, Aristotle categorized motion into four types.

- Alternation (Chemistry)

  Alternate motion was really the motion of chemical change. Burning and decay were the examples explained in the first idea.

- Natural local (Vertical) motion

  Because each of the four earthly elements had its natural place, vertical motion was explained by Aristotle as the natural motion to put each object in its natural position. (Earth below water, water below air, air below fire and all below ether.)

- Horizontal (violent) motion

  Horizontal motion was treated as governed by something different than natural vertical motion. To state it simply, Aristotle would have said that to move something you have to push it. The pushing is a more modest was of saying that you had to do violence to it. Objects inherently did not want to move and required external forces to create and maintain horizontal motion.

- Celestial (perfect circular) motion

  The last category of motion was celestial motion and it was different is a special way from all the others. It required no violence or external force. The heavens naturally moved in circles because circles were perfect and the heavens were perfect and the special material of the heavens, ether, provided no inhibition to circular motion.

Our interests are in mechanical motion, but this will eventually explain horizontal, vertical and celestial motion as a consequence of one fundamental set of laws. Chemical change requires almost three hundred more years of study to understand. It was only at the beginning of the twentieth century that atoms were generally accepted as the basic constituent of matter and only then could chemistry really begin to be explained according to the laws of physics. That will be our sixth idea - the quantum theory.

2.1 Projectile Motion al la Aristotle

Aristotle was not one to let facts ruin a good theory. There was one fact, which was no explicable in the simple terms of natural and violent motion. That was the explanation of what happens when you throw something.

When you throw something it arcs into the air, goes up for a while and the falls. Our modern term for this is projectile motion. Projectile motion is nothing more than the motion exhibited every time you throw a rock..

The Aristotelian theory says that throwing shouldn't work very well. Remember, horizontal motion is violent motion and to make something move you have to push it. Well, what's pushing the rock after you throw it?

Aristotle had to make something up especially for the case of projectile motion. It is called "antiperistasis". The meaning of this is that as an object move through the air, it leaves empty space behind it. Empty space, in the view of the ancients, could not exist. (This concept was later immortalized in the phase "Nature abhors a vacuum." That's another theory by proclamation, but we won't go into that now.) Given that no one has ever seen a vacuum, it sounds reasonable that Nature must abhor it, so the air behind an object must rush in violently to fill the space vacated by the moving rock. In fact, and this is the key to the Aristotelian explanation, the rushing air moves so forcefully that it ends up pushing the rock!

The antiperistasis explanation is pretty forceful and complex, so no one asked for a long time how it works with narrow sharp objects like a spear. A spear, especially one thrown backwards, would be pushed forward by the air behind it only if the air pushed on the trailing point. It is sort of hard to imagine air pushing on the point of a spear. Fortunately for Aristotle, a man of his reputation did not have to defend his theory against every objection of every ignorant common man. Indeed, later (for about a thousand years) if you did not accept and endorse the Aristotelian view of things, that was an indication that you must be some ignorant common man.

We should begin to appreciate the appeal of logic and objectivity a little more when we realize that profoundly now obviously incorrect doctrines in science were believed for a thousand years by force of social imposition. One does not need to appeal to the sins of modern man (e.g. genocide) to see that whole societies may be made to act and apparently believe grossly wrong things as conventional knowledge. Who knows what things we believe today might be also "great lies".

3 Galileo's Studies of Falling Objects

Around 1600, Galileo's interests in mechanics led him to perform some simple experiments. He was interested in the laws which governed the motion of falling objects because his observation of everyday objects were not in accord with the doctrine of Aristotle's followers. It took years, but he found that he could slow the motion of falling objects by rolling balls down an inclined plane. This overcame an essential problem, namely that things which fall do not normally do so with enough time to make careful quantitative observations.

The logic he employed in studying balls rolling down inclined planes should be appreciated as an important step to making his discoveries about the laws of motion of objects. It is probably fair to assume that as a casual observer he noticed that heavier objects generally fall at the same speed as light ones. Also, things seem to fall fasted the longer they are allowed to fall. Neither of these observations were consistent with Aristotle. Essentially, the belief of the time was that one you let go of a object, it instantaneously acquired a velocity and moved towards earth. How fast it did this depended on its weight.

The genius that Galileo showed by using balls rolling down inclined planes as a substitute for falling objects should not be underestimated. He had to believe that by introducing two new qualities of the motion, namely the circular motion of rolling and the delaying effects of horizontal motion couple to the vertical motion when moving down the plane, did not alter the fundamental law governing the vertical motion.

Leaning Tower of Piza experiments were probably not actually done. We know of this experiment because it was discussed as a hypothetical experiment in one of his books.

4 Galileo's Inclined Plane Results

- Inclined plane slowed motion so it could be observed.
- Acceleration and Deceleration and the Law of Inertia

  - When rolling up hill, object slowed.
  - When rolling down hill, object speeded up.
  - When rolling on flat surface objects maintained speed.

- Heavier bodies do not fall faster.
- Uniform motion means distance is proportional to time
- Uniform acceleration means distance is proportional to square of time.
- Galileo understood the role of
  - experimental error.
  - air resistance and friction.
  - terminal velocity.
- Projectile motion is the superposition of horizontal and vertical motion.

From his inclined plane results, Galileo almost formulated Newton's laws of motion. If Galileo had been able to integrate the concept of force with motion, he would have anticipated Newton by half a century.

5 Air Track Class Experiments

In class, an air track was used the illustrate the motion of a glider executing almost frictionless motion. This is a somewhat modernized version of Galileo's experiments with the inclined plane.

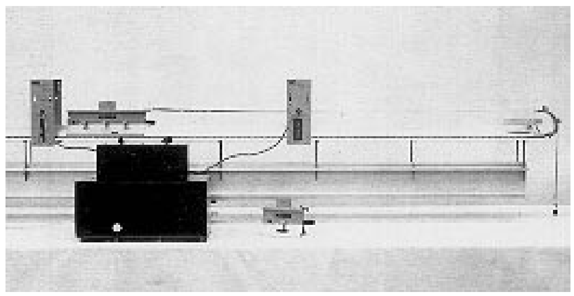

Figure 1: A picture of someone's air track.

Our demonstration were divided into *uniform motion* where an object moved along at a constant speed when the track was level and *uniformly accelerated motion* when the tract was tilted.

5.1 Uniform Motion

We observed what Galileo observed. There is a special orientation of the air track that we call "level". In that orientation an object placed on the (nearly) frictionless track simply sits there. If given an initial push to the left, it travel left at a seemingly uniform rate. If pushed initially to the right, it continues without decay of the motion to the right.

For uniform motion,

1. Acceleration is zero.

2. Velocity is constant.

3. Straight line distance increases linearly with time.

All three items above are equivalent statements. Uniform motion may be defined as straight line motion at a constant velocity.

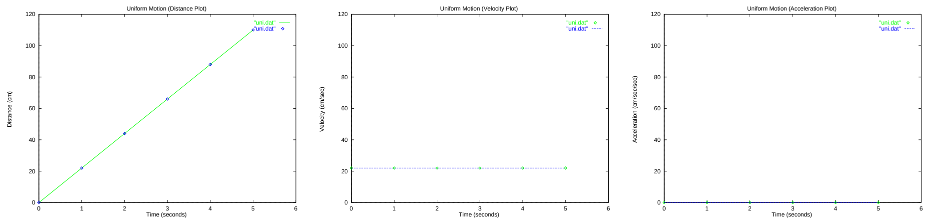

Figure 2: Distance, velocity and acceleration for uniform motion.

We plotted the distance, velocity and acceleration of the air cart, as shown in Fig. 2.

The straight line graph for distance versus time is a graphical representation of the well known equation,

$$d = vt\,,$$

which says *distance equals velocity times time.*

In the particular example from the class experiment, we determined that the cart moved 1 meter in 4.5 seconds, or with a velocity of 22 centimeters per second which we write as 22 cm/sec.

> The **steepness** of the distance versus time graph reflects the velocity. This is the same as saying the **slope** of the graph gives the velocity.

The above observation is clear for a simple straight line but it is actually a much more general statement, true for any distance versus time graph.

The velocity graph for uniform motion is completely flat. The height of the line is, of course, equal to the velocity. Because of this simple shape, it is a simple observation that the area under the velocity line between the time = 0 and some later time = t is the length (t) multiplied by the height (v). This area is $d = vt$. This allows us to say something here which is actually true even under more complicated circumstances of variable velocities.

> The **area** under a segment of a velocity versus time graph is equal to the distance covered in that time.

5.2 Uniform Accelerated Motion

The modern version of Galileo's inclined plane experiment is carried out with an inclined air track, Fig. 1, where the nearly frictionless surface results in uniformly accelerated motion. Supposing one carried out the measurements carefully with an incline of only two and one half centimeters (2.5 cm) from one end to the other of a two and a half meter (250 cm) air track, such a one per cent (1%) rise would result in an acceleration of about ten centimeters per second per second (10 cm/sec/sec).

Fig. 3 shows the results that one would get for the inclined air track experiment if the measurements were done extremely accurately.

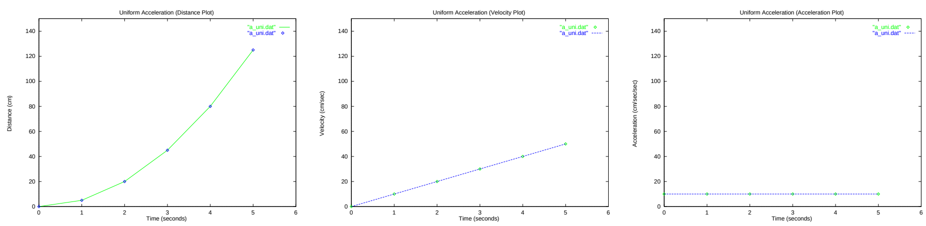

Figure 3: Distance, velocity and acceleration for uniformly accelerated motion.

We observe , as did Galileo, that the distance increases as the square of the time. Mathematically we would say, $d \propto t^2$. The symbol, $\propto$, means "proportional to". The mathematical curve for distance in the accelerated motion figure, Fig. 3, is a parabola. Interestingly, that shape is one of the conic sections that is permitted as a motion of the planets. Parabola's occur frequently in nature.

Looking at the graph of velocity versus time, we see that it is linear. The shape is the same as the shape of the distance plot in the uniform motion plots in Fig. 2. The slope of the velocity plot is the change in velocity each second.

> The **acceleration** of an object is defined to be the change in velocity per second.

Of course, we can define a new work any time we want, but the laws of nature may or may not choose to use our definitions. In fact, nature does use it. The fact that the plot of velocity in the inclined plane experiment is a straight line mean the acceleration is uniform. **Uniform** means only that it is the same thoughout the experiment.

> The **acceleration** of an object caused by gravity as it descends along a straight inclined plane is constant.