Idea 3: The Energy Concept - The Power of the Machine

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October 9, 1999

1 Preamble: Energy Powers Mechanics

Discovering the great mechanical universe gave us the laws of the great machine, but did not tell us what makes it go!

2 Vis Viva: Overview

We discovered in the last idea that there was something conserved in collisions, namely momentum. We will review that and give an overview of the concepts of mechanical energy before we go into great details.

1. Conservation of Momentum

   The momentum of an object is its mass time velocity. since velocity is a vector, so is momentum. It is conserved when objects interact (and when they don't). Conserved means that the total amount of momentum never changes. After any set of events, if you do the accounting properly and consider momentum of all participants and all forms of momentum, then you always have the same moment as when you started.

   Total momentum of a set of objects is the sum of the mass times velocity of each individual object.

   Total momentum is conserved in a collision.

   (a) Force causes acceleration inversely to the mass of an object

(b) Acceleration changes velocity, hence momentum.

(c) If the mass of one object is larger than another, then if the objects have the velocity, the large mass has the larger momentum.

(d) The same force when applied to two different object for the same time results in a change in momentum of the objects that is identical.

(e) by Newton's third law, the forces acting on two objects during collisions are equal and opposite.

(f) two objects of different masses will thus change velocity by different amounts, but the mass time velocity change of the two objects will be exactly opposite.

(g) The changes in the mass time velocity of two objects cancels out when you add up the total momentum after a collision.

(h) momentum is conserved in collisions.

2. Elastic Collisions

   In an elastic collision all of the force (work) has been used to change the state of motion of an object and none of the force (work) has gone into the permanent deformation of the object.

   For example, two air carts collide when a spring is placed between them and they bounce apart.

   Counter example, two air carts collide but have magnets attached to the collision point so they stick together.

3. Vis Viva

   In an elastic collision, something other than mass time velocity, mv, is conserved. Through experimentation, in Newton's time it was discovered that sometimes mass times velocity squared, mv^2, was conserved. Experiments showed that **elastic collisions conserved vis viva**.

   Today, we know this conservation property by a different name. We say kinetic energy (energy an object has by virtue or its motion) is conserved and we write the definition of kinetic energy, KE, as

$$KE = \frac{1}{2}mv^2 \, , .$$

4. Work

   Work occurs when a force acts on an object and the object moves in the direction of the applied force.

   The equation that defines work is

$$W = Fd\,. \tag{1}$$

   We say then that work done by a constant force, "F" acting on an object that moves a distance "d" does an amount of work equal to F times d. In this equation, it is important to point out that we only count the amount of force pointing in the direction the object moves.

   When the force is not constant, there must be more complicated formulae than Eq. 1 developed but that's where calculus comes in. Rather than spend a lot of time on these special formula, we can show that the area under a force versus distance graph is the work done on the object and by the force.

5. Work = Change in KE

   Kinetic energy is energy by virtue of motion. We can show (Sec. 4) that the work done by a force accelerating an object is exactly equal to the change in kinetic energy of the object. As an equation, this reads,

$$W = KE\,.$$

6. Potential Energy

   Mechanical Energy, i.e. all that one might encounter in the study of forces and motions of objects, includes another important type of energy, **potential energy**. As the name implies, this is the potential to do work.

   (a) Gravitational Potential Energy

   PE due to the position of an object in a gravitational field. Normally, we consider the Earth's field, but that is just a special case that we encounter often enough to write a special formula for it. If an object has a potential to fall a distance "h", then its weight is the force that acts through this distance. The weight of an object is its mass times the gravitational acceleration,

$$W = mg\,.$$

Since work is force times distance and work increases the kinetic energy, we can say that the potential energy of an object a height h in the Earth's gravitational field is

$$PE = mgh\,.$$

(b) Springs and Potential Energy Springs occur often enough in mechanics that we have a special *law* for the called **Hook's Law**. Hook's law simply says that a spring stretched for a distance we will call "x" exerts a force proportional to "x". As an equation, we write,

$$F = -kx\,. \tag{2}$$

The force is proportional to the distance stretched. That's Hook law. The amount of force depends on the type of spring. In fact, every spring is different. So, to know the force, you need to know (usually by measuring) the proportionality constant or, as we call it in physics, the *spring constant.* The minus sign in Eq. 2 specifies that the force exerted by the spring is always int he opposite direction it is stretched. If stretched, the force tends to make it shorter, but if compressed a spring exerts a force to make itself longer.

When we use a force due to a spring, we are not using a constant force, so any work we can do with a spring will not be simply given by Eq. 1. In the case of a simple spring, the force increase with distance and the work increases with distances in such a way that the work a compressed (or stretched) spring is capable of doing is proportional tot he square of the distance. Using calculus or determining the area under a curve of spring force versus distance, we can arrive at the formula

$$PE = -\frac{1}{2}kx^2\,.$$

Our **conclusion** will be after studying the interchange of work, potential energy and kinetic energy that there is a principle of conservation of mechanical energy. We will have to distinguish what happens when frictional forces are present (both inside and outside of an object), but when they are absent, as in so-called **elastic collisions**, we can saw we have a conservation law.

In a mechanical system with no losses due to friction, total kinetic plus potential energy remains constant.

$$KE + PE = \text{Constant}$$

3 Conservation of Momentum

The basic conservation laws of mechanics include conservation of energy and momentum. The momentum conservation law is truly universal. No violation of this law has ever been observed. When, in the course of discoveries in physics, it appeared that perhaps momentum might not be conserved, scientist were led to deeper understandings of nature and always, in the end, momentum was still conserved.

3.1 Mathematics of Momentum

If two objects collide, then Newton's third law says that they experience equal and opposite forces. Let's look at a simple case when that force of interaction is a constant, call it "F" and the duration of the collision is a time interval, call it "t".

With a constant force, acting on a mass "m", Newton's second law says

$$F = ma\,.$$

If the Force is constant, then the velocity increases linearly with time and

$$v = at\,.$$

If we simply multiply the left sides of these equations together and the right sides together, the result is

$$Ft = m(at) = mv\,.$$

The result is that a constant force acting for a time "t" causes a momentum that is exactly the mass time velocity.

> More precisely, the result of a net force acting on an object produces an increase in momentum equal to the force time the time of application and the direction of the added momentum is in the direction of the force.

4 Mathematics of Work and Kinetic Energy

In Sec. 3.1, we used mathematics to show that a constant force accelerated an object and added a momentum, mv, to the object that was exactly equal to Ft. This fact, plus Newton's third law is a mathematical proof that mechanical momentum must be conserved in a collision.

A similar proof exists for work and energy in a collision. Let's look at the same collision as in Sec. 3.1, but calculate now the work, Fd.

A constant force, F, acting on a mass, m, for a time, t, will move it a distance, d. Work, is $F \times d$, so let's determine what d is, assuming the mass starts at rest.

As before, we know F and m, so we can determine acceleration using,

$$F = ma\,.$$

After a time, t, the object acquires a velocity v,

$$v = at\,.$$

The *average* velocity during the time t was only half of this because it started at zero and ended at v.

$$\bar{v} = \frac{1}{2}at\,.$$

The distance, d, is, as always, the average velocity times the time,

$$d = \frac{1}{2}at^2\,.$$

Now that we know the distance, we can calculate the work done by a constant force, $W = Fd$, and relate that to the mass and velocity. Multiply the equation for F by the equation for d and we get,

$$Fd = (ma)(\frac{1}{2}at^2) = \frac{1}{2}m(at)^2\,.$$

To express this in terms of velocity, we replace the occurrences of at by v. Then, we see that

$$Fd = \frac{1}{2}mv^2\,,$$

or

$$W = KE\,.$$

We conclude that an impressed force F for a distance d results in a kinetic energy equal to the work done, Fd. In a collision, the force between two objects is equal, by the distance through which the forces act in not. If the body deforms, then one body may deform more than another. The deformation is part of the distance. If it is an "elastic body" where the deformation requires a force and the exact same magnitude force is present when the object returns to its original shape, then the forces and distances all work out so that energy is conserved in an "elastic collision".

> We now have a definition of an **elastic collision**, one where the action of the forces between the bodies goes entirely into changing the kinetic energy of the body and not into the effects of the deformation of the body.

5 Heat - The Other Form of Energy

Many of the basic laws of physics aren't known to young children, but the **Mechanical Equivalent of Heat** is. Everyone knows that if their hands are cold, they rub them together to make them warm. To understand this is a deeper scientific way, we must first understand the difference between temperature and heat. Then, we can quantify heat and make a precise connection between heat energy and mechanical energy that we have studied thus far. Our discovery process will follow the historical process and begin with our old friend Aristotle who got this wrong too.

1. Aristotle

   Fire was one of the basic elements of nature. To Aristotle is was just something that existed. We humans knew about it when we it came into our bodies (or left our bodies). Then, we felt hot or cold. Hot, for Aristotle, was just feeling the element fire.

2. Phlogiston

   By the 1600s, Galileo and his contemporaries still believed in the basic materialism of heat, but it was somewhat more elaborate than Aristotle.

   (a) Heat was a fluid with mass.

   (b) Heat could be driven out of objects. Smashing or distorting objects was one way. Burning was another.

   (c) Different object were understood to be able to hold different amounts of heat.

   (d) Heat was something that lived within a body, and because it was released when the object burned or was destroyed, was viewed as being the "soul" of matter.

   (e) Amounts of heat, and other properties like its "soul", were not particularly quantified.

3. Caloric Theory

   Developed by Antoine Lavoisier in the late 1700. Lavoisier is know as the father of chemistry. He was among the first to use the periodic table to discover elements.

(a) The "caloric" was a fluid with no mass.

Lavoisier investigated this by weighing substances that had been heated and those which were cold. No differences could be detected.

(b) Caloric was invisible.

(c) Caloric fluid flowed from hot to cold.

(d) A quantitative measure of caloric was established.

In modern terms this is the **calorie**. We will return to this in Sec. 7.

6 Steam Engines

(a) Newcomer Engine (1712)

The power of steam to do work was known for a thousand years and examples can be found in Chinese history. In western history, Thomas Newcomer created a steam engine in 1712 to use to pump water out of coal mines.

The work of the Newcomer engine was done entirely by the process of condensing steam which creates a partial vacuum on a chamber. That partial vacuum was used to move a piston and thereby accomplish work.

(b) Watt Engine (1769)

James Watt developed a more sophisticated and much more efficient engine in 1769. Watt's improvements were on both engineering and design. His engine operated at a higher pressure and did work on both expansion and compression. He, most notably, included a separate condenser for the steam.

Watt's main goal was making money and he did this very well. Watt's contemporaries understood that efficiency was a key to success, but there remained fundamental misunderstanding about how to achieve that efficiency because there remained fundamental misunderstanding of the nature of heat and work.

(c) Sadi Carnot (1824)

Carnot was an engineer who wanted to explain from a theoretical point of view the process of changing heat into work. He developed a "water wheel analogy" which worked well and ultimately led to the fundamental understanding of a new law of physics (but that is Idea 4).

7 Quantifying Heat

We now appreciate the importance of heat in a way that scientists did a couple of hundred years ago, but we (and they) have not been very quantitative. Modern folks in daily life confuse **temperature** and **heat**, and this reflect the need for clarity that exists in the 1700s.

7.1 Temperature

7.1.1 Thermometers

Thermometers measure temperature, but how does one know that the concept of "temperature" makes any sense? Physiologically we can say things are hot and cold in a relative sense. In an absolute sense, there is a problem. Two people in the same room will disagree one whether the room is hot or cold.

The human sense of temperature is pretty good for comparing two objects in the sense that two different people will generally agree whether two different object "feel" hot or cold. Unfortunately, these same two sets of feelings will disagree with a thermometer.

Imagine that a person walks into a room that has a normal temperature and see a book and a steel ball lying on the table. If the person touches the book, it feels neutral or warm to the touch. If the person touched the steel ball, usually they will say it feels cold. If you take your thermometer out of the medicine cabinet and touch it to the book and the ball, the thermometer will read the same temperature for both? What gives?

The human sense of temperature does not measure temperature at all. It is a better sensor for how fast heat flows from body tissues to the external world. When you touch the book or the steal ball, your fingers loose some of their heat to the object touched. The reason the ball feel cooler is that more heat flows from your finger to the ball than to the book.

Why is that? Read on.

7.1.2 The Direction of Heat Flow

If we consult the average eight-year-old ask if heat flows from hot to cold, they will tell us *yes.* They will be correct and they will have the basic understanding needed for the concept of entropy that is Idea 4. If you ask the same eight-year-old if cold flows from cold to hot, you may get the same affirmative answer. There lies the problem. What is the stuff we are discussing hot or cold? Are they different?

The correct answer is that there is only heat and lack of heat. It is not a fruitful scientific avenue to treat cold as an entity. We can only really demonstrate this convincingly after we understand **absolute temperature**, a concept which occurred a century or so after the understanding that temperature was a concept separate from heat. (Later, we will cover this issue.)

The fundamental understanding we need to delineate here is the understanding that *heat flows from hot to cold*. We can perform an imaginary (or real) experiment. Suppose you have two identical objects (balls or books, it doesn't matter.) Suppose they are at different temperatures and separated from each other and somehow separated from the environment. (They may be wrapped in insulation to keep their temperature from changing or it may be that we do not need to wrap them if we do our experiments quickly enough to that not much heat enters the air.) We say that an object is **thermally isolated** if it is separated from its environment so that it doesn't change in temperature. Thermal isolation is never perfect, but like the elimination of air resistance as part of Galileo's discovery of the mechanics of motion, being mostly eliminated is enough for our discovery too.

One we have prepared out experiment with separated objects of different temperatures, we can allow them to touch. We say then that we have placed them in **thermal contact**. If the two identical objects at different temperatures are brought together, we can just touch them ourselves and tell that after a while the warm one gets cooler and the cold one gets warmer and in the end they both have the same sensation of temperature. This characteristic is a fundamental requirement of any method for defining or measuring temperature.

generalizing the above experiment to any old objects made of any material and of any temperature initially, we arrive at the basic law of thermometry.

> Two objects at different temperatures, when brought into thermal contact, will reach the same temperature.

Any method of establishing temperature must follows this basic law of thermometry. Fortunately, the early methods worked well and the advances in the understanding of heat progressed rapidly one we understood temperature. Among the early forms of thermometers were types similar to the thermometers we have in out medicine cabinets today. There were glass tubes with columns of liquid that rose or fell depending on the temperature. While the history of thermometry is fascinating, we do not have time to follow it much further here.

7.2 Heat Capacity

Once some idea of temperature existed, the basic questions of heat fell into place. For instance, how much heat can an object hold and how is that heat related to temperature of the object.

> The **heat capacity** of an object is the amount of heat that it absorbs when being raised by one degree in temperature.

We need to define one unit of heat. We will call this a calorie.

> A **calorie** is the amount of heat it takes to raise one gram of water one degree Centigrade.

With this definition, every object we encounter can be labeled with a heat capacity. One needs simply have a method of supplying heat and then compare the amount of heat absorbed to the number of grams of water that could be heated one degree with the same amount of heat. Fortunately, thermal properties of matter depend on the type of matter and not on its shape or locations or lots of other things. Thus, one gram of gold will take the same amount of heat to raise it one degree whether it is in the shape of a ear ring or a coin. This leads to the concept of specific heat.

> The **specific heat** of a material is the amount of heat that *one gram* of the material absorbs when being raised by one degree in temperature.

Even more convenient is the property of most materials to have a specific heat that is nearly the same at all temperature. That allow us to write an equation for the quantitative change in the amount of heat an object has as its temperature changes.

Let us call C the specific heat of a certain material. (Gold, iron, water, air, people all have different values of the specific heat. This is something that is normal determined by experiment.) To raise a mass of m grams of the material an amount in temperature ΔT, we need to ad an amount of heat, H, given by

$$H = Cm\Delta T\,.$$

One can look up the values of heat capacity, C, in books for different materials. having those numbers, you can tell how much heat it takes to heat a body. Conversely, it tell how much heat can be extracted form a body by allowing it to cool.

7.3 Thermal Conductivity

Every object also has a property that tells how fast heat can travel in it. That property is thermal conductivity and has a lot to do with the physiological sensation of temperature.

7.4 Latent Heat of Vaporization

It takes one calorie of heat to raise one gram of water one degree - until it boils. At the boiling point, suddenly an enormous amount of energy is absorbed and no change in temperature occurs. This amount of heat that it takes to boil a gram of material is called its **Latent Heat of Vaporization.**

8 Mechanical Equivalent of Heat

The caloric their explained a lot of things, but failed to apply the scientific method rigorously and got a few things wrong.

(a) The caloric went into the spaces in matter, thereby explaining thermal expansion - except for water and ice.

(b) Friction generated heat by rubbing small pieces of material off of a body and breaking up the cavities inside the body thereby letting the caloric leak out.

(c) In 1807 Guy-Lussac experimented with expanding gasses and found that expanding gasses did not get cooler even though the spaces in the material must be getting large and could therefore hold more caloric.

(d) In the caloric theory, caloric was conserved and could not be created.