Idea 6: Quantum Mechanics: Richness in Indeterminism

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1 Preamble: Quantum Mechanics

Since Newton's fundamental work on mechanics in the 1600's[1], there followed over two and a half centuries of more discoveries of nature laws and a continuing confidence in science's ability to know and predict everything. By the early 1900's, with Special Relativity in the bag and chemistry coming on as the science of the century everything looked rosy. Determinism reigned supreme.

By the middle of the 1920's, something was clearly wrong. A rather small problem of predicting the amount of heat energy radiating from objects - nothing more complicated than the glowing embers of a fire or the bright filament of an incandescent light - was proving to be the undoing of many long held theories. Within a decade, great upheavals of science occurred and quantum mechanics was born.

The quantum theory says that the laws of nature are not deterministic. Hence, all previous theories of physics, were wrong. (Arrgh!) The 20s-40s saw the development of quantum mechanics and the restructuring of thermodynamics, statistical mechanics and electricity and magnetism.

We now understand that determinism is fine up to a point. Just as Newton's laws are completely adequate for normal every day life and we do not need to know special relativity to drive to the grocery store, the same is true for modifications of most of the basic theories (e.g. mechanics, electricity and magnetism) to account for the loss of determinism. We do not need to abandon the deterministic approach to science, we just have to limit its use to the time and space domains where nondeterministic effects are very small.

With that conforting thought, we can go on to Sec. 2 learn just what this new theory of Quantum mechanics really is about.

2 Origins of Quantum Theory

As electromagnetic theory was advancing to its crescendo in the mid 1800s, those researcher in electromagnetism and optics were finding tremendous success with the applications of formal mathematics to physics problems. Among the natural phenomena they were investigating was light emitted from bodies that were heated. They thought that this phenomena provided an intriguing connection between the properties of matter and the properties of electromagnetic theory and light. Unfortunately, the did not have a clue as to the theory to describe why and how heated bodies emitted light.

Remember, apart from the Sun, light from heated or burning objects was the only light people had back then. Even now, the way a simple light bulb works is by emitting light from a tungsten filament heated to thousands of degrees. So, this was not some weird backwater area of science, but a major unexplained phenomenum.

2.1 Black Body Radiation

All objects give off heat radiation. This was not understood very well at the beginning of the century, but this was what needed to be investigated. Just as we discovered in the study of mechanics, to understand the true laws of physics, Galileo and later Newton and to remove complicating effects like friction and focus thier attention of the basic phenomena under study. We saw the need to simplify in the study of thermodynamics too. We created *open* and *closed* idealized systems to discover the laws of thermodynamics. The idea of a black body is the idealization needed to help researchers study heated and light radiated from hot objects. Without getting technical, it means just what you think, an object that is black and does not reflect light.

A black body is very special. In physics we apply that term to an object which is perfectly absorbing of all light that hits it. This is true not only in the visible wavelengths, but in the very long wavelengths (infrared and below) and ultraviolet wavelengths.

From conservation of energy, a black body must give off energy too. It does this by radiating that energy - again as light. The puzzle that early researchers were trying to solve was how does the radiation that a black body gives off depend on wavelength and how does that energy depend on temperature.

From our common experience, we know something about black body radiation. We have all observed the embers of a fire. We know that when we see the red glow, it means that part of the burned wood is still hot. You may have also observed that if a fireplace tool is left in the hottest part of a fire then, when it is pulled out, it may be glowing almost white. These are examples of light emitted from bodies that are hot. From our experiences, we know that the hotter objects are "white hot" and that as they cool, they become "red hot".

2.2 Black Body Radiation from a Fireplace

It is not so easy to connect our personal experience with electromagnetic radiation outside of the visible wavelengths. Visible light occupies the wavelength region between about 3000 and 7000 Angstroms. That range is about a millionth of an inch. Longer wavelengths, such as microwaves that cook your food, can not be seen and shorter wavelengths such as x-rays are equally invisible.

The heat from a fire is mostly in the long wavelength region, visible and microwave. You can detect the microwave energy as a sense of heat. The next time you are next to a fire place, notice that when you open the door or screen to the fireplace you get a sudden sense of increased warmth. If you hold up your and, a couple of feet from the fire, you can feel the heat on the side facing the fire and there is little or no sense of heat on the side away from the fire. If it was the air that was warm, both sides of your hand would feel about the same. It is because the "heat" is actually long wavelength electromagnetic radiation. It feels like heat only after it is absorbed by your skin and converted to kinetic energy of the molecules of your body.

These observations that we may make around a fireplace tell us the essential qualities of black body radiation. An object will radiate energy at all wavelengths, but the greater amounts are radiated at long wavelengths. The amount radiated at shorter wavelengths (the violet end) increases as temperature increases. Fig. 1 shows a graph of the amount of energy radiated as a function of wavelength at two different temperatures. As ca be seen, temperatures of thousands of degrees are necessary to generate much energy in the visible range.

The mathematical description of how much energy is radiated and how it depends on temperature and wavelength was posed as a problem in 1859 by Gustav Kirchhoff who was an internationally known physicist at the time. The solution to the problem would wait until 1900 for solution. That 1900 solution would ultimately give Max Planck the Nobel prize in 1918 for discovering the first elements of quantum theory.

3 Planck's Quantum Invented

Max Planck invented quantum mechanics on a rainy Sunday afternoon after a visit from a fellow physicist named Rubens who had been working on the problem of black body radiation. Planck later said that when working on the problem of predicting the equation for this radiation he based his predictions on some known properties of classical antenna's. Planck assumed that matter was made of small bits of charge that oscillated in all possible frequencies. This idea was not new, but he added a twist. He guessed that the problems with previous theories could be removed if each tine oscillating part of matter were assumed to have specific allowed energy levels. If the energy that one of the oscillating bits was quantized, then an embarrassing infinity in previous theories would go away. He worked on this for several hours and decided that if he wanted to get the right answer then he would have to put in a constant that was adjusted to match the experiment. He called the constant **h**. We now call this **Planck's constant.** It is one of those fundamental constants of nature that we can not predict.

Planck's fundamental contribution to quantum mechanics was based on a guess. He guessed that electric charges in matter would oscillate with discrete amounts of energy proportional to the frequency of oscillation. He guessed,

$$E = hf \, .$$

This is the h that we now call Planck's constant. Planck was not even going to publish his result because he could not justify this guess except to say that it worked. He assumed that if it was correct, eventually he or someone else would arrive at this equation and figure out what the constant was. Fortunately, one of his colleagues convinced him that it was worth publishing, even if he did not fully understand why it was true. In 1918, he would get the Nobel prize in physics for this work which started on a rainy Sunday afternoon.

4 Photo Electric Effect

The photo electric effect is again, a fairly simple phenomena. But, it was not understood, so scientists wanted to know why it happened. One of those scientists was Albert Einstein.

The photo electric effect occurs when light shines on a metal. The electrons in that metal can be kicked off by the light. In particular, the higher the frequency of the light, the more energy they have when they are kicked off. The very first recorded observation of this is by Heinrich Hertz in 1888. These are the basics facts that Einstein had (in rather greater detail, of course) when he thought about this in 1905.

If one looks at the formula from Planck, $E = hf$, and looks at the energy of the electrons that get knocked off of a metal by light, one finds that there is a one-to-one relation. It was known that to get an electron off of a metal required a certain minimum energy. We call that the "work function". It means what it implies, it is the work required to remove an electron. Now, if you give the electron more energy than is required to just get it off, it comes off with some kinetic energy too. What was observed was that kinetic energy increased linearly with frequency,

$$\mathrm{KE} = \mathrm{hf} - \mathrm{Work}\,.$$

This formula is peculiar because it says that the light waves can give only a quantized amount of energy to the electrons. Better yet, it says that the light acts as though it made up of bundles of energy, each bundle being an amount hf . This would be OK for particles, but it is heresy for waves. No waves are every seen to have a certain fixed amount of energy. That is a property normally associated only with particles.

Einstein said that whether waves with specific amounts of energy were seen or not, this what the experiments were saying. Einstein said that light can apparently be viewed as being composed of a lot of individual light quanta and each light quanta had an energy hf . This was perhaps the first time a wave was attributed to having the properties of a particle. It could collide and transfer energy and momentum just like a particle. When we speak of light acting like a particle, we speak of it as a large composed of a number of light particles called **photons.** Each photon has an amount of energy given by E=hf.

That was the beginning of the wave-particle duality. That means that attributing the properties of a particle to some things and the properties of a wave to other things is really not right. What we have learned since then is that all things have wave properties and also have particle properties. For instance, particles are normally associated with collisions and wave are associated with phenomena like diffraction. You would never expect a particle to diffract or a wave to impart momentum in a collision that way a particle does. The understand of how these things happen requires understanding quantum mechanics and its inherent indeterminism.

5 Quantum Calendar

- 1890 Atoms existence promoted by Boltzmann.
- 1896 Electron discovered by J. J. Thompson - Plum Pudding model of matter.
- 1900 Quantum of energy hypothesized by Planck to explain black body radiation.
- 1907 Photo-electric effect explained by Einstein.
- 1912 Rutherford discovered hard core of atoms, Plums disappear and planetary model appears.
- 1913 Neils Bohr conjectures wave interference as “allowed orbits”. Hydrogen worked.
- 1916 Sommerfeld added relativity to Bohr’s model. Worked better.
- 1924 Louis de Broglie proposes that particles have a wave character.
- 1925 Heisenberg proposes a quantum theory and uncertainty relations. (Correct.)
- 1927 Davisson and Germer experimentally see that particles have a wave character.
- 1926 Erwin Schrödinger publishes wave equation for particles. Modern QM discovered.
- 1928 P. A. M. Dirac makes Schrödinger’s theory relativistic.

- 1948 R. P. Feynman describes Quantum Electrodynamics (E and M plus Relativity.)

6 Schrödinger's Equation

Schrödinger arrived at his equation after reading the thesis of de Broglie. In that thesis, de Broglie had theorized that if waves could have particle properties, then perhaps particles could have wave properties. He was quite specific saying that it was the momentum of the particle that determined the wavelength and it was related by Planck's constant. In particular, de Broglie conjectured that wavelength was inversely proportional to momentum according to,

$$\lambda = \frac{h}{p}\,. \tag{1}$$

Schrödinger, Einstein and many others were fascinated by this proposal that de Broglie made. To suggest that particles could have wave properties was very radical. After reading about this idea, Schrödinger gave a little seminar on it to the students in the college where he taught. One of the students suggested that if particles were to exhibit wave properties, then there should be a wave equation to describe them. Schrödinger was well aware of waves in many other areas of physics and knew that in some simple instance the de Broglie relation (Eq. 1) did correspond to a wave equation. He had to generalize this to a case when forces were exerted on the particle. He did this by seeing that there was a correspondence between energy conservation and de Broglie's wave condition.

The equation which Schrödinger arrived at and which now bears his name is

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = E\Psi\,. \tag{2}$$

Today, the Schrödinger equation is used in most areas of physics. The wave function, Ψ, describes the wave proerties of the particles in a quantum system. While all systems are really quantum systems, the effects of quantum mechanics are really large and important in dealing with atoms. In fact, one of the greate motivations for trying to develope the quantum theory was that explaining the spectrum of light from atoms was a total failure with classical physics.

7 Planetary Atom Model

While the Schrödinger equation is now universally accepted, it turns out that Schrödinger does not get credit for actually inventing quantum mechanics. He was second. Werner Heisenberg was also working on the problem of predicting the spectrum of light from atoms. The ideas that preceeded him were that electrons went around the nucleus of the atom in much the same way that planets went around the sun. This **planetary model** was only inveneted in 1912 after Rutherford had done experiments shooting aluminum atoms with helium atoms. (Helium was the result of radioactive decay of large atoms and natually occuring radioactivity was the only way they had of making atoms move extremely fast.) With his experiments, Rotherford discovered that atoms were mostly empty. Atoms had 99.9% of their mass concentrated at their center.

The planetary model was the opposite of what had been believed since J. J. Thompson had discovered the electron in 1896. Since electrons were part of matter and enectrons were negative, they reasoned that the rest of matter must be positive. Since all they knew about to cause forces between **subatomic particles** was Coulomb's law, they assumed that the positive and negative parts of matter would just stick together in a blob. That idea became known as the **Plum pudding model.** The picture they had of atoms was some kind of mushy soft postive material and distributed in it were the negative electrons - like plums in a pudding. They had never seen positive particles, so they assumed the positive stuff was soft and mushy.

When Rutherford's experiments blew the plum pudding idea out of the water, it was very easy to imagine an atom with a heavy center and orbiting electrons. After all, the law of gravity and Coulombs law were mathematically similar, both inverse square laws. Scientists immediately started writing theories about small electrons orbiting a heavy highly charged central nucleous. This brought some good news and some bad news. Electromagnetic theory was completely clear that a charge moving in a circle would radiate light. Well, that was good, becuase we knew that atoms gave off light. What was bad was that the light should be borad spectrum (contrary to the discrete spectra observed) and the charges should continue to give off light while the electrons spiraled inward toward the nucleous and never stop until the electrtons actually banged into the center.

Of course, the problem here is that the electrons are being described as particles. We had to discover how to describe them as waves.

8 Bohr's Model of an Electron Wave

Before the equations of quantum mechanics were written down and before de Broglie had conjectured that particles were waves, there was another wave idea that was used to explain light from atoms.

9 Heisenberg Uncertainty Relations

10 Schrödinger's Cat and Indeterminism

References

[1] I. Newton, *Principia Mathematica*

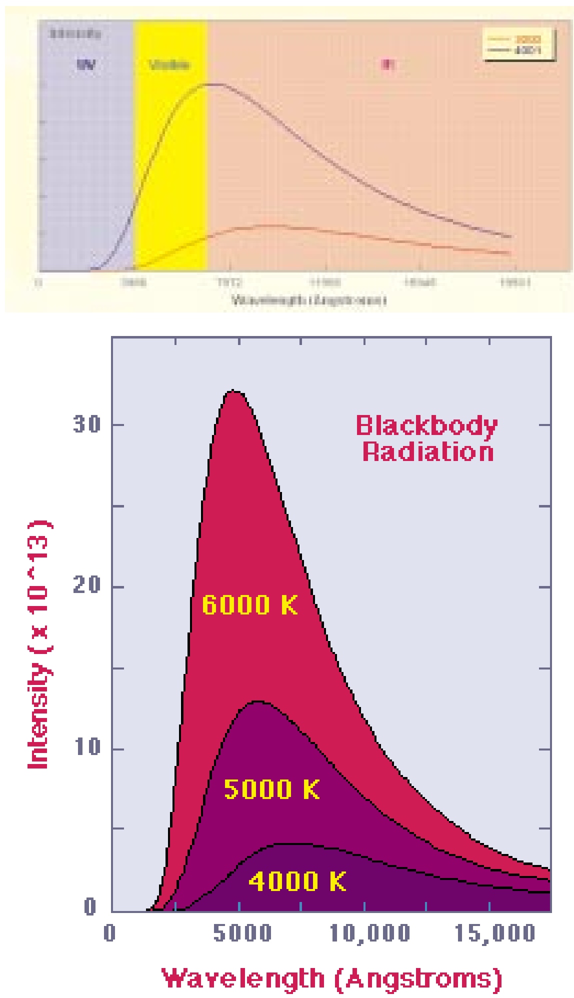

Figure 1: Plot of Black Body Energy Emission

Figure 2: Max Planck

Figure 3: Erin Schroedinger

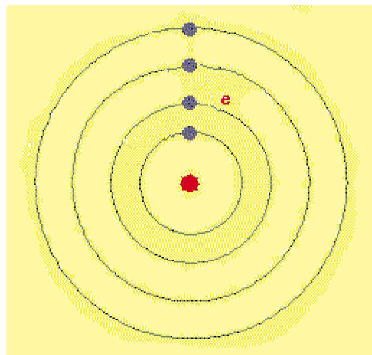

Figure 4: Planetary Model of an Atom

Figure 5: Neils Bohr

Figure 6: Werner Heisenberg

Figure 7: Schroedinger's Cat